

# RELIABILITY MODELING VIA DATA ANALYSIS

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## Abstract

Realistic system reliability models must include many factors of known or potential importance. Such models will contain many parameters, which should be estimated from failure data. This seldom will be feasible since there will be too few failures to provide reasonably accurate parameter estimates. Another approach is to analyze existing data sets, to help determine reliability drivers. This paper concentrates on the analysis of two well known failure data sets, or more accurately, two too well known sets-of-numbers, Proschan's (1963, 2000) aircraft air conditioner data and Davis' (1952) bus engine data. The practical implications of the data sets differ drastically from the incorrect implications of the misinterpreted numbers and some of the correct implications, which have been virtually ignored for decades, are obvious even from "eyeball analysis." Statistical analysis of repairable systems failure data has great potential for improving our understanding of system reliability drivers. To achieve this potential, however, it is essential to consider the fundamental differences between parts and repairable systems – and hence, the corresponding differences between their probabilistic models – in analyses.

## 1. Introduction

This paper shows that important maintenance insights often can be obtained using very elementary methods, sometimes even by "eyeball analysis." Simply recognizing that nonrepairable items (henceforth, parts) are discarded after they fail, but systems usually are repaired after their failures, leads to different probabilistic models, and more importantly, to different maintenance strategies. Moreover, important differences hold even in the "most special" cases of the exponential distribution for parts and the homogeneous Poisson process (HPP) for repairable systems (henceforth, systems).

## 2. Preliminaries

The following material is presented in much greater detail in Ascher and Feingold (1984, pp. 7 – 46).

### 2.1 Model for nonrepairable items (parts)

The underlying reliability model for parts is:  $R(x) = \Pr\{X > x\}$  where  $R(x)$  is the reliability function,  $X$  is the random variable, time to part failure, and  $x$  is the real variable, part operating time.

#### 2.1.1 Force of mortality (FOM)

Assuming that  $R(x)$  has a probability density function (PDF),  $f(x)$ , the FOM,  $h(x)$ , is defined as  $h(x) = f(x) / R(x)$ . Probabilistically,  $h(x)$  is best interpreted as (Meeker and Escobar, 1998), "the propensity to fail in the next small interval of time, given survival to  $x$ ."

Statistically, very little can be done with a single time to failure so  $h(x)$  is estimated in terms of the ratio of total failures in an interval to the total part-hours of exposure to the risk of failing in that interval. Hence, it is essential to emphasize that probabilistically, i.e., the way it is defined,  $h(x)$  is a property of a single time to failure.

### 2.1.2 Wearout and burn-in of parts

For the purposes of this paper we will say that parts wearout (burn-in) if their underlying reliability function has monotonically increasing (decreasing) FOM.

## 2.2 Model for repairable systems

Since a repairable system in general suffers two or more failures, its underlying model is a stochastic point process, i.e., the model is a sequence of times between failures, henceforth called interarrival times. Down times for repairs, which usually are very short compared to interarrival times (and which were not recorded in the two data sets considered in this paper), will be ignored throughout. The only specific stochastic point process model considered in this paper is the HPP, which can be defined as a nonterminating sequence of independent and identically exponentially distributed interarrival times.

### 2.2.1 Rate of occurrence of failures (ROCOF)

Assume that a system has been observed over an operating interval  $[0, t]$  and that it has failed  $N(t)$  times in that interval. Then the ROCOF,  $v(t)$ , is defined as  $v(t) = d/dt\{E[N(t)]\}$ , where  $E[\cdot]$  denotes expectation, assumed to be differentiable. It is well known that the HPP has a constant ROCOF, which is numerically equal to the constant FOM of each of its exponential interarrival times. Section 2.3 shows, however, that even in these most special cases, FOM and ROCOF have up-to-infinitely different interpretations.

### 2.2.2 Deterioration and improvement of systems

For the purposes of this paper, we will say that a system deteriorates (improves) if its successive interarrival times become stochastically smaller (larger).

## 2.3 FOM versus ROCOF

When the area under a FOM increases to infinity, the reliability function,  $R(x)$ , decreases to 0. That is, when the area under a FOM increases to infinity, one failure will occur, with probability 1. In infinite contrast, when the area under a ROCOF increases to infinity, the expected number of failures increases to infinity. The fact that FOM and ROCOF can be – and sometimes, must be – represented by the same mathematical function, makes it imperative to avoid using the term “failure rate” for either one since then “failure rate” ends up being used for both, interchangeably. It is equally essential to avoid using “t for two” different time scales. The infinite difference in interpretation of FOM and ROCOF applies to the most special cases of the constant FOM of an exponential distribution and the HPP’s constant ROCOF, just as for other FOM’s and ROCOF’s.

## 3 Analysis of Proschan’s aircraft air conditioner (A/C) data

### 3.1 The data

The A/C failure data tabulated by Proschan in 1963, and reprinted in 2000, are the best known set of reliability data. Proschan's (1963, 2000) Table 1 presents 213 interarrival times between failures of 13 A/C's. The Table indicates when each of four A/C's received "major overhauls", once each. The after overhaul data were treated as if they were data from four additional A/C's, for a total of 17 data sets.

### **3.2 Proschan's analysis**

The a priori assumption of system renewal by each repair is usually grossly unrealistic, but often is still being made. Contrary to some claims in the literature, Proschan tested for renewal in his Section 3, rather than assuming renewal a priori. Using a trend test based on ranks, he found no evidence of trend, for any of the data sets or for the results pooled over the 17 data sets. In his Section 4, Proschan showed that each of the 17 data sets could be modeled by exponentially distributed interarrival times, i.e., each data set was modeled by an HPP. Section 5 invoked the theorem that a mixture of exponential distributions has decreasing FOM to show that the parameter of the exponential interarrival times, and hence of the HPP modeling each data set, varied among the data sets. Section 6 discussed the implications of the mixture of exponential distributions theorem, but only for parts, rather than for the A/C's and other systems.

### **3.3 Physical interpretation for parts of the mixture theorem**

Assume that the 213 numbers in Proschan's Table 1 are the times to failure of 17 batches of parts. The FOM is constant for each batch but varies among the batches so the parts with the larger FOM's tend to fail first. No individual part is getting more (or less) reliable but the survivors are more reliable than the 213 originally put into operation. Because failed parts are discarded there is survival-of-the-fittest parts, i.e., the parts are burning-in.

### **3.4 Physical interpretation for A/C's of the mixture theorem**

Proschan's analysis showed that the ROCOF was constant for each of the 17 data sets but that it varied among the data sets. Since each A/C was always repaired to its prefailure ROCOF, including the one with the largest ROCOF, there was no improvement either individually or collectively. Put another way, for as long as each data set can be modeled by an HPP, there will be no improvement even after the A/C's have "frozen hell over!"

### **3.5 Physical differences correspond to different probabilistic models**

Parts with large FOM's usually fail early and are screened out of the group originally put into operation. In contrast, even the least reliable A/C always was repaired and returned to service just as unreliable as before its failures. This obvious physical difference corresponds to a major difference in the underlying probabilistic models: A mixture of exponential distributions, with different FOM's has decreasing FOM but the superposition of independent HPP's is an HPP, whether or not the superposed HPP's have equal ROCOF's. Since the Table 1 data are from repaired A/C's, the mixture theorem implies heterogeneity among the A/C's constant ROCOF's, rather than decreasing FOM in any sense.

### **3.6 Analogy between implications for parts and systems**

If we can identify the A/C or A/C's with high ROCOF's, then discarding them corresponds to discarding failed parts. However, identifying poor A/C's statistically will take considerable time and

there always will be positive probability of misclassifying a poor (good) A/C as good (poor). Moreover, it might not be economically feasible to discard an A/C which could be repaired and returned to service. A potential alternative to discarding poor A/C's is to improve them. The Table 1 data show that improvement might be difficult to achieve.

### **3.7 Pessimistic implications of A/C overhauls**

Four A/C's were overhauled, once each, during the observation interval displayed in Proschan's Table 1. It is obvious that the A/C on Plane 7908 was worse after its overhaul (of the 13 interarrival times before overhaul, 7 were 100 hours or more whereas none of the 10 after overhaul was in triple digits). In addition, there is some indication that 7911 deteriorated after overhaul. (It is almost as obvious that 7909 was better after its overhaul.) If "major overhauls" did not improve 3 of 4 A/C's, and actually deteriorated 1 or 2 of them, how could the A/C's be improved, short of major redesign?

It is noteworthy that the overhauls often have been ignored in tabulations of these data. Even worse, the overhauls often have been indicated but then their implications usually have been ignored!

### **3.8 Summary**

Even "eyeball analysis" of Proschan's Table 1 provides maintenance insights which have been overlooked / disregarded for forty years. Even though Table 1 is the best known set-of-numbers in the reliability field it remains a very poorly understood and analyzed data set.

## **4 Analysis of Davis' bus engine data**

### **4.1 The data**

Davis' (1952) data have not been cited as often as Proschan's but they also often have been misinterpreted, see Ascher and Feingold (1984, pp. 147-148). The data were reproduced as five histograms by Ascher and Feingold (1984, p. 87). The histograms display 191 miles to first failure, and 105, 101, 96, 94 interarrival miles to second, third, fourth and fifth failures, respectively. All censored data, which must have been extensive, were ignored. Two important things are equally apparent from eyeball analysis. First, the later histograms tend towards an exponential shape: in particular, the last histogram, with 12 bars, is almost monotone non-increasing. Secondly, the successive histograms cluster closer to the origin; the sample means are 94,000, 70,000, 54,000, 41,000, and 33,000 miles, respectively.

### **4.2 Caveat about data**

Easterling's (1985) review of Ascher and Feingold (1984) pointed out that the trend towards shorter interarrival miles might have been an artifact of the extensive data censoring, instead of showing real bus engine deterioration. The larger an interarrival mileage, the more likely that failure would not have been observed by the analysis cutoff point. He had an excellent point and it certainly would have been very useful to have more complete data, at least all of the censored data available at the time of Davis' (1952) analysis and preferably the complete histories of all 191 engines. It is noteworthy that no other reviewer or colleague has made this important observation.

The lack of censored data has another important implication. Not only will the successive mean interarrival times be underestimated, in addition, the tendency towards exponentiality might be delayed or eliminated. The shorter an interarrival time, the more likely that it will be fully observed

rather than censored, leading to a monotonically decreasing estimate of the underlying PDF. Moreover, after the large decrease from 191 engines run to first failure, to 105 engines observed to have failed twice, the later small sample size decreases are still accompanied by substantial reductions in the sample means. For example, with a decrease of only 2% in sample size between the fourth and fifth failures, the sample mean still decreased by 20%.

Henceforth, the only available data will be taken at face value, i.e., it will be assumed that successive interarrival miles were becoming more exponential and also getting stochastically shorter.

## 4.2 Interpretation of data

Regardless of the obvious merits of having more complete data to analyze, Davis' (1952) bus engine data, in their highly censored form, have been a major source of the claim for widespread – if not almost universal – “exponentiality.” However, the exponentiality exhibited by the later histograms, when accompanied by the shorter and shorter means, has a diametrically opposite interpretation than does exponentiality of parts! It is well known that part exponentiality implies that preventive maintenance (PM) should not be performed since it would be ineffective. In contrast, when a sequence of interarrival times (or miles) with decreasing means reaches an unacceptable level, exponentiality suggests that PM should not be performed – because too much maintenance is needed! Since exponentiality suggests that there are many important failure modes, engine replacement also should be considered.

## 4.3 Summary

Analysis of more complete data might dilute or eliminate the apparent deterioration – and/or the apparent tendency towards exponentiality. However, even “eyeball analysis” of the available data provides maintenance insights which have been overlooked or disregarded for fifty years.

## 5 Overall Summary

Easterling (1985) observed, “The world is full of important repairable systems, and it is crucial for our understanding of them that we collect, analyze, and learn from data in a sensible and useful way.” Amen.

## References

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